Thought: Powerball Lottery Tickets Are Actually Worth Less Now That The Jackpot Prize Is Higher

1/14/16
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ThoughtBurner

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UPDATE: I found an error in the calculations of the original post (1/12/16). This version has the corrected probability calculations, figures, and numbers. The general interpretation is the same, but the specifics changed. You can still download a copy of the old version at the end of this post.

You might want to think twice about buying a Powerball ticket.

The Powerball jackpot prize is currently a whopping $1.5 billion, making it the **largest jackpot lottery ever**[i]. The media hype is huge, and people all over the country are **lining up**[ii] to have a chance to become unthinkably rich overnight. If there was ever a good time to buy a lottery ticket, it would be now, **right**[iii]?

Well, no.

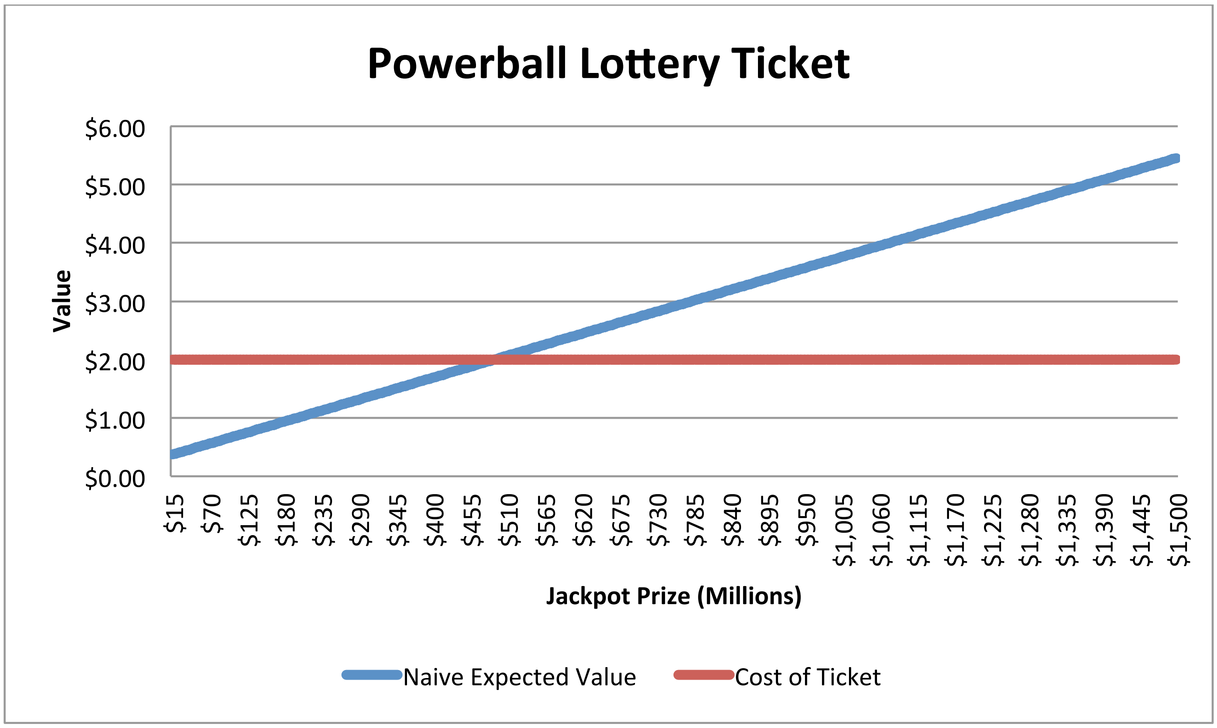
Many people don't seem to realize that a higher jackpot doesn't necessarily increase the value of a lottery ticket. Sure, the total jackpot prize gets higher. But as more and more people buy tickets, *the odds that somebody else will also win the lottery increases significantly*. This is bad because splitting the lottery with someone else greatly reduces the size of the jackpot payout, and therefore makes the lottery ticket much less valuable.

So, when should you buy a lottery ticket? The key idea is that you want the jackpot to be high enough to make the ticket worth the cost, but not so high as to induce a lot of people to buy tickets – you've got to find the jackpot sweet spot. When trying to find this sweet spot, you need to adjust the expected values of lottery tickets to reflect the changing probability of having multiple winners – that is, of having to split your winnings with someone else.

The Naïve Expected Value Approach

The standard approach used to determine whether it is "worth it" to buy a lottery ticket is to use an expected value calculation (for example, see **this**[iv] and **this**[v]). The basic idea is: find the "expected value" (EV) of the lottery ticket, which is simply the probability of winning a prize times the dollar amount of that prize, and then compare that to the cost of the ticket. Since there are multiple potential prizes, each of these possible prizes are multiplied by their respective odds and added together to determine the lottery ticket's expected value. If the expected value is greater than the cost of the ticket ($1 for Mega Millions and $2 for Powerball), then you should buy a lottery ticket. If the expected value of the ticket is lower than the cost, then you should save your money for something better. Odds can be found for **Powerball**[vi] and **Mega Millions**[vii] on their websites.

The trick to this technique is that the expected value of a lottery ticket increases as the jackpot increases. This is because as the dollar amount of the prize increases, the price of the ticket and the chance of you choosing the winning numbers stay the same. In the end, it all comes down to finding the minimum jackpot value that makes the expected value of a lottery ticket equal to its cost. In the case of this fairly straightforward calculation, that comes out to about $491 million. Below, you can see the naïve expected values for Powerball tickets plotted over jackpot size, along with the cost of a ticket:



The point where the two lines cross show where the jackpot prize makes the ticket 'worth' its cost. The higher the jackpot prize, the more the ticket is worth. According to this calculation, the $1.4 billion Powerball prize indeed means that lottery tickets are more valuable than ever, worth an expected $5.11.

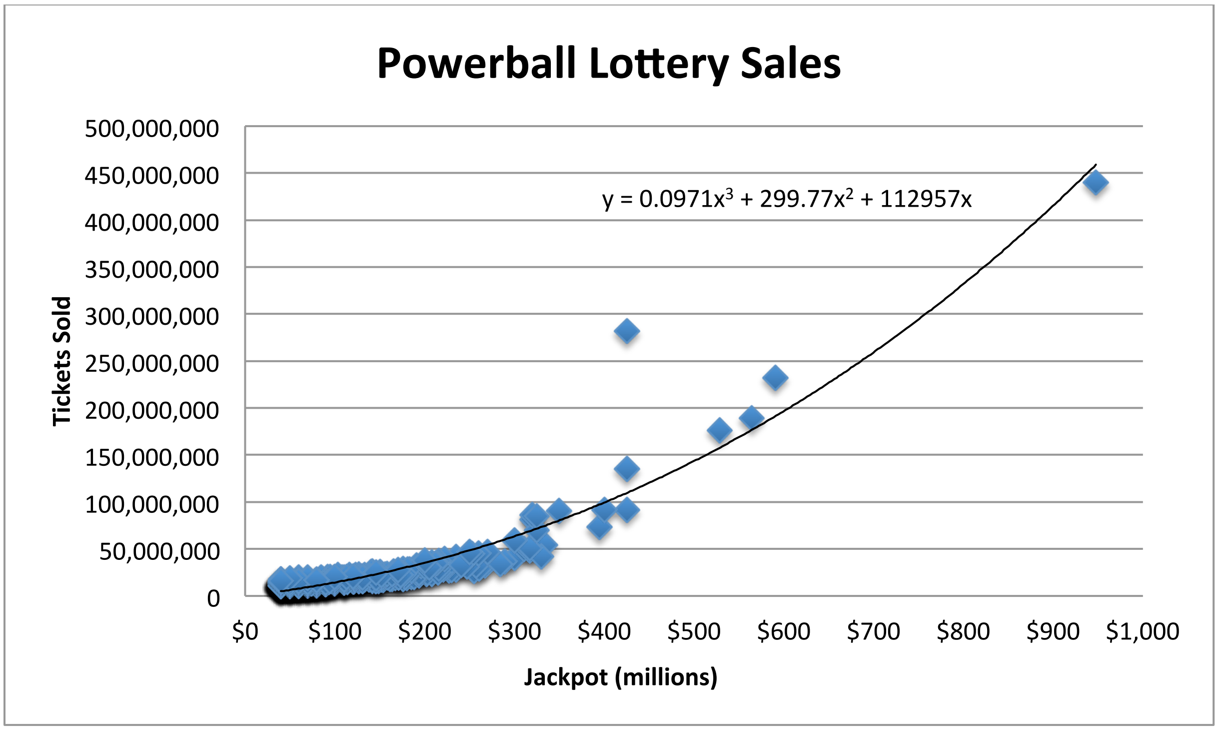
Splitting the Jackpot

This standard approach, however, doesn't take into account the fact that as the jackpot amount gets higher and higher, the number of tickets sold increases. This doesn't change *your* odds – you still have the same chance of winning the lottery – but it does increase the chances that *someone else will also win the lottery*. This makes a big difference in the expected value of a lottery ticket: if two people both choose the winning numbers, they have to split the jackpot, effectively cutting the value of winning in half. If three people win, they have to split it into thirds… and so on and so forth.

This is why lottery tickets don't necessarily keep increasing in value as the jackpot prize amount increases. Depending on how many more people buy lottery tickets as the prize increases, the value of lottery tickets could fall. You want the jackpot to be high, but not so high that you'll have to split the winnings with someone else.

In order to take this into account, I used **lottery ticket sales data**[viii] to predict the number of tickets sold depending on the Powerball prize. This will help us adjust the expected values based on how likely it is that you'd split the jackpot with some other lucky person (or unlucky person, given that you'd both have to share now).

Below, I've plotted ticket sales over the size of the jackpot prize for every Powerball drawing since they switched to a $2 ticket (January 18th, 2012). The trend line is a cubic function fitted to the sales data. It estimates the number of tickets sold (*y*) based on the size of the jackpot prize in millions (*x*).



Using the equation from the model to estimate the number of tickets sold for each jackpot prize, I then calculated the odds that, given you have already won the lottery, you would have to split it with someone[1]. For example, with a $300 million Powerball prize, the model estimates that about 63,000,000 tickets will be sold. This makes the odds that nobody else wins equal to about 80%. If the prize were $600 million, however, the odds that nobody else will win the lottery is only about 51%. The odds that nobody else will win the jackpot decreases as the prize increases because the number of estimated tickets sold increases as the prize increases.

[1] I actually calculated the odds for each possibility – that nobody wins, that exactly one person wins, that exactly two people win, etc. – and added the expected value of each outcome together, until the expected values were too small to make a difference (i.e. less $0.01).

For each of the odds of splitting the jackpot, I multiplied the probability of the outcome by the prize amount you would receive if the outcome actually occurred. For example, if there was a $100 million prize with a 70% chance of *not* splitting the lottery, then the adjusted expected value of winning the lottery ticket changes from $0.34 to $0.29:

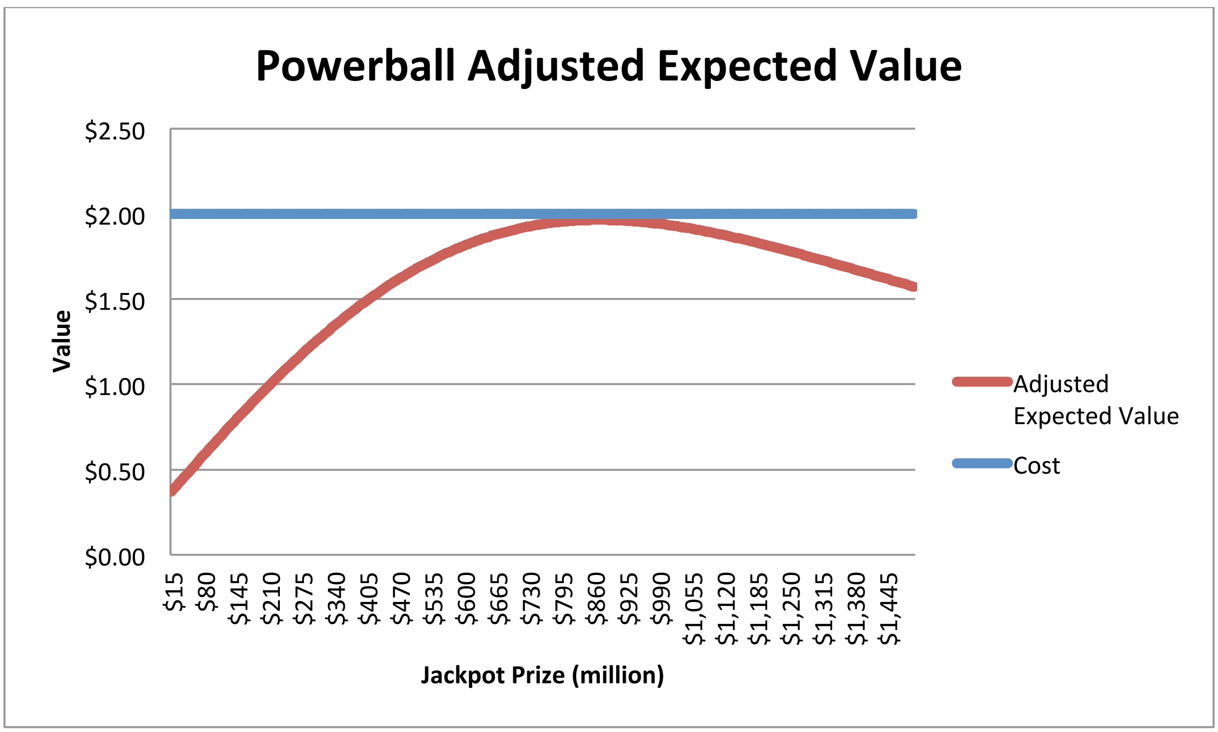
$$Adjusted\ EV\ of\ Jackpot\ Win = (P\ of\ Winning\ Jackpot) * (70\% * \$100m + 30\% * \left(\frac{\$100m}{2}\right))$$

$$Adjusted\ EV\ of\ Jackpot\ Win = (0.000000003422) * (\$85m) \approx \$0.29$$

$$\boldsymbol{UN}adjusted\ EV\ of\ Jackpot\ Win = (0.000000003422) * (\$100m) \approx \$0.34$$

Notice that this adjusted expected value of winning the jackpot is *less than* the unadjusted expected value of winning the jackpot. This will always be true because this new expected value takes into account the increased probability of having to split the jackpot prize as its amount increases. For each jackpot, I calculated the adjusted expected value of winning. Then, I added the expected value of the other prizes[2], and the result is the new adjusted expected value of the entire Powerball ticket.

Below, I've plotted the adjusted expected value of a Powerball ticket, along with its cost.



As you can see, at first, the expected value of lottery tickets rises as the jackpot prize increases. But eventually it hits a tipping point where the ticket's expected value actually starts to decline. This is

[2] The expected value of the other, non-jackpot prizes remains the same, since these aren't split if there are multiple winners.

because with a higher jackpot prize, more people buy tickets, which increases the probability of having to split the prize so much that it actually makes the ticket less valuable than it was before.

Specifically, the graph shows that Powerball tickets are most valuable when the jackpot is between $870-$875 million, where they have an expected value of about $1.96. At this jackpot size, the prize is high enough to increase the expected value of the ticket a good amount, but not so high as to make it very likely that you would have to split the jackpot with other people (about a 49% chance you have to split it with someone else).

Notice that the maximum expected value of a Powerball ticket is always less than the cost of a ticket ($2). This just reinforces the idea that buying lottery tickets is a horrible investment strategy. If you were to play Powerball an infinite number of times (infinite drawings, not infinite tickets), there is no strategy that will let you win money. And even if you bought 5 tickets for every Powerball drawing for 100 years, you would still have 99.98% chance of never choosing the winning ticket.

While the $1.5 billion dollar prize may be the largest jackpot in history, the adjusted expected value of a Powerball ticket right now is not actually at it's maximum. In fact, the current expected value of a Powerball ticket is approximately $1.57 – which is about the same as the expected value of a Powerball ticket if the jackpot were $445 million.

This is all just to say that now is in fact *not* the time to buy Powerball tickets. That time was last week on Saturday, January 9th, when the jackpot would have paid out $950 million and Powerball tickets had an adjusted expected value of $1.96 – which is just $0.01 short of the maximum. I'm sorry to say, you've all missed the sweet spot already.

Then again, it is 1.5 billion dollars.

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[i] http://money.cnn.com/2016/01/06/luxury/largest-lottery-jackpots/
[ii] http://www.wsj.com/articles/powerball-no-winner-so-jackpot-may-hit-1-3-billion-1452410989
[iii] http://www.marketwatch.com/story/you-should-really-buy-a-700-million-powerball-ticket-today-2016-01-08
[iv] http://time.com/money/4172196/powerball-math-odds-advantage/
[v] http://davidtorbert.com/2012/03/is-it-ever-worth-it-to-play-mega-millions/
[vi] http://www.powerball.com/powerball/pb_howtoplay.asp
[vii] http://www.megamillions.com/how-to-play
[viii] http://www.lottoreport.com/salescomparison.htm